Problem Solving – march 2020_1 - Solutions

Some Useful Information

Prime numbers less than 1000

2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113	127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199	211	223	227	229	233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349	353	359	367	373	379	383	389	397	401	409
419	421	431	433	439	443	449	457	461	463	467	479	487	491	499	503	509	521	523	541
547	557	563	569	571	577	587	593	599	601	607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733	739	743	751	757	761	769	773	787	797	809
811	821	823	827	829	839	853	857	859	863	877	881	883	887	907	911	919	929	937	941
947	953	967	971	977	983	991	997												

There are ***25*** primes less than 100; ***62*** primes less than 300; ***95*** less than 500; ***168*** less than 1000.

Practice Meet Problems (Actual Meet Problems from Prior Years)

[1] A bowlful of strawberries was on a picnic table at a barbecue. First, Ramon ate 1/5 of the strawberries in the bowl. Next, Sidney ate 1/3 of the strawberries that were left. Later, Ramon helped himself to more strawberries, this time eating ¼ of the strawberries left in the bowl. Next, Dad ate 50% of the remaining strawberries. Finally, Mom ate 2/3 of what was left, leaving 2 strawberries in the bowl. How many strawberries did Ramon eat altogether? ***10*** *(6+4)*

[2] In the game of Parcheesi, in order for your piece to leave "Home" you must get a 5 on one or both dice, or the sum of the dice must equal 5. What is the likelihood that your piece will leave home as a result of your first roll of the dice? (Assume fair six-sided dice numbered from 1 to 6.) Express your answer as a fraction in simplest terms. ***5/12***

[3] 35 children were out on the playground during a recent winter day. They were all dressed in winter jackets but some were wearing other winter accessories as well. 12 of them wore either gloves or mittens and a hat with their jackets. 3 of them wore snow boots and jackets only. 3 were bundled in gloves and mittens, hats and snow boots. 2 wore only their winter jackets out to play. 5 of the children wore gloves or mittens only in additions to their jackets. 4 wore no mittens or snow boots but wore hats. Twice as many children wore hats only with their snow boots as wore gloves or mittens only with their snow boots. How many children were dressed in a jacket with a hat and snow boots only? ***4***

[4] In the multiplication problem below, different letters represent different digits. What two-digit number does AB represent? ***75***

```
        A  B
X       A  B
_____________
    C  A  B
B  D  B
_____________
B  E  D  B
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[5] Farah is fascinated by the triangular numbers (1,3,6,10,15,21, and so on). Recently she came across a clock and found that she could rearrange the twelve numbers 1,2,3,…12 around the face so that each adjacent pair added up to a triangular number. She left the twelve in its usual place.

What number did she put where the 6 would usually be? ***5*** *(Sequence:12,9,6,4,11,10,**5**,1,2,8,7,3)*

[6] A dartboard has only two regions: one worth 9 points and the other 4 points. If you could throw any number of darts, what is the largest score that you cannot get? ***23 points***

[7] Create a 10-digit number so that the first digit tells you how many 0's are in the number, the second digit tells you how many 1's are in the number, the third digit tells you how many 2's are in the number, etc. ***6210001000***

___ ___ ___ ___ ___ ___ ___ ___ ___ ___

0's 1's 2's 3's 4's 5's 6's 7's 8's 9's

[8] There are 300 students in the 9th grade and each student is assigned a locker. The 300 students start down the hall one at a time. The first student opens every locker. The second student closes all the lockers that are multiples of 2. The third student changes the lockers that are multiples of 3 (which means if a locker is closed, they open it. If the locker is open, they close it.) The fourth student changes all the lockers that are multiples of 4. This continues until the 300th student changes the 300th locker. Then, the principal walks down the hall, and changes all the lockers that are prime numbers.

(a) At the end, how many lockers will be closed? ***221*** *(17 squares, 62 primes)*

(b) In another school there are 1000 ninth graders. If they did the same activity as above, how many lockers would be open at the end? ***199*** *(31 squares, 168 primes)*

[This is the classic locker problem made exceedingly difficult by adding the principal changing all prime number lockers. There is no calculation for determining the number of primes.]